Exam Answers Labour Economics, Spring 2014

June 27, 2014

Part I - Shorter questions (max 1200 words)

Question A

Example answer: The answer depends very much on whether human capital is general or specific, that is whether the human capital accumulated in one job can also be used in other jobs.

If human capital is general, wages will increase with human capital because as workers accumulate more human capital they become more productive and attractive for other firms and so will require a higher wage to stay with their current firm (their outside option improves). In this case, Daniel's wage will increase faster than Nikolaj's wage because he accumulates human capital faster and we will expect Daniel to have a higher wage than Nikolaj (at some point) in the future.

If human capital is specific and cannot be transferred from one job to the next, wages will not necessarily increase with human capital because workers outside option does not change when they accumulate human capital (although the wage may still increase if wages are set in a way that gives the worker bargaining power, e.g. Nash Bargaining). If the wage does not increase with human capital we would expect the two identical workers to earn exactly the same, both today and in the future (although see the caveat regarding search frictions below).

Finally, returning to the case where wages increase with human capital and focusing on today's wages, we would most likely expect Nikolaj to have a higher current wage than Daniel. To see why this is the case, note that since the wage in Daniel's job increases faster over time, Daniel's job is strictly more attractive if the initial wage is the same. In order for Nikolaj to be willing to accept a job with a slower wage growth than Daniel's it must therefore be the case that Nikolaj's job starts out with a higher wage. The only exception to this is if the labor market is characterized by search frictions. In this case it is possible that Nikolaj just randomly encountered and accepted a strictly worse job offer than Daniel, despite the fact that they are identical workers. This would be an example of frictional wage dispersion (in lifetime wages).

Question B

Example answer: Lowering the income tax rate by one percentage point in 2010 increases the after tax wage by some number of percent in each of the countries (the exact number depends on what the initial tax rate is). Let g denote the percentage increase caused in the after tax wage in Gondor and m denote the percentage increase in the after tax wage in Mordor. If we assume that the governments simply burn their tax revenue, the two tax reforms do not have any other effects on people's incomes or consumption of public goods so the only thing changing in 2010 are the wages workers face.

If we first look at Gondor, the tax reform in 2010 changed the after tax wage only in that one year but did not change the wages in any year after 2010. This corresponds to what is called a temporary or transitory wage increase. The labor supply response to such a shock is often (approximately) measured by the Frisch elasticity, which formally is defined as the percentage increase in current labor supply of a one percent increase in the current (after tax) wage, keeping marginal utility of wealth (or income) constant. Since labor supply in Gondor increased by 2 percent in 2010 in response to a g percent temporary increase in the (after tax) wage, the Frisch labor supply elasticity in Gondor is $\frac{2}{g}$.

Looking next at Mordor, the tax reform in 2010 caused a change in the after tax wage both in this year and all subsequent years. This corresponds to a permanent wage increase or an increase in the overall wage profile. The labor supply response to such a shock is measured by the long run labor supply elasticity: the percentage increase in labor supply that occurs in response to a permanent one percent increase in the (after tax) wage. Since labor supply in Mordor increased by 0.5 percent in 2010 in response to a permanent m percent increase in the wage, the long run labor supply elasticity in Mordor $\frac{0.5}{m}$.

Now we note that the difference between the Frisch elasticity and the long run elasticity occurs because a temporary wage has (approximately) no income effects: when the wage only increases in one year, it has practically no effect on your lifetime income. Conversely, when the wage increases permanently, you will earn more at all future dates even if you do not change your labor supply, which implies that there is a important increase in lifetime income. Assuming as usual that leisure is a normal good, a permanent wage increase implies a negative income effect on labor supply: As people feel richer, they will tend to work less. As a result, the Frisch elasticity is larger than the long run elasticity.

Based on the above, we can therefore make the following statements about what would happen if we flip the tax reforms: The number of hours worked in Gondor would increase by g times the long run labor supply elasticity in Gondor. This elasticity is smaller than the Frisch elasticity, which in Gondor is $\frac{2}{g}$, so the increase in number of hours in 2010 would be less than 2 percent. The number of hours worked in Mordor would increase by m times the Frisch labor supply elasticity in Mordor. This elasticity is greater than the the long run elasticity, which in Mordor is $\frac{0.5}{m}$, so the increase in number of hours in 2010 would be more than 0.5 percent.

Part II - A job search model (max 1100 words)

Question A

Example answer: If we let the wage distribution be given by $H(w)$ and the job arrival rate as unemployed and employed, respectively, be given by λ_u and λ_e, we know that the reservation wage is given by

$$x = z + (\lambda_u - \lambda_e) \int_x^{\bar{w}} \frac{1 - H(w)}{r + q + \lambda_e [1 - H(w)]} dw$$

but since here $\lambda_u = \lambda_e = \lambda(m/n)$, then $x = z$. It means that workers are indifferent between working and being unemployed at the unemployment income level when they get the same frequency of job offers.

Question B

Example answer: The model generates wage dispersion because as shown by Burdett and Mortensen (1998) the Diamond critique is overcome due to on-the-job search. It is enough to argue that at least one firm will pay more than the wage $w = z$ as in the Diamond critique. To begin with, consider the case where all firms pay the wage $w = z$. Then, a firm offering $w = z + \varepsilon$, where ε is an arbitrarily small number, can attract all workers, so no worker will ever turn down an offer from this firm. Likewise, no workers will leave the firm besides due to exogenous job destruction. Thereby, the firm will in steady-state have more workers than the other firms paying the wage $w = z$ and since $(y - w)$ is almost the same when ε is small enough, the firm offering the wage $w = z + \varepsilon$ will earn strictly positive profits whereas all other firms earn zero profits.

Question C

Example answer: In steady-state the inflow into unemployment equals the outflow from unemployment that is

$$\begin{aligned} n(1-u)q &= nu\lambda(m/n)(1 - H(x)) \Leftrightarrow \\ nu &= \frac{qn}{q + \lambda(m/n)} \end{aligned}$$

where we have set $H(x)$ to zero as no firms will post wages below the common reservation threshold x since it will never hire workers and, hence, it will earn negative profits. Furthermore, u is the unemployment rate and nu is the unemployment in terms of persons.

Question D

Example answer: When we derive $\ell(w)$, we need to take account of the fact that the number of workers, n, and firms, m, are not of unit mass as in the Cahuc and Zylberberg textbook version of the Burdett-Mortensen model and we more or less follow the derivation in the published article, Burdett and Mortensen (1998). First, we calculate the equilibrium wage distribution function $G(w)$ by equating outflow of workers from this mass with the corresponding inflow.

$$\begin{aligned}
nu\lambda(m/n)H(w) &= n(1-u)G(w)[q+\lambda(m/n)(1-H(w))] \Leftrightarrow \\
G(w) &= \frac{u}{1-u}\frac{\lambda(m/n)H(w)}{[q+\lambda(m/n)(1-H(w))]} \Leftrightarrow \\
G(w) &= \frac{qH(w)}{[q+\lambda(m/n)(1-H(w))]}
\end{aligned}$$

where we have used that $u = \frac{q}{q+\lambda(m/n)}$. Differentiating $G(w)$ delivers

$$\begin{aligned}
g(w) &= \frac{qh(w)[q+\lambda(m/n)(1-H(w))] + qH(w)\lambda(m/n)h(w)}{[q+\lambda(m/n)(1-H(w))]^2} \\
&= \frac{q[q+\lambda(m/n)]h(w)}{[q+\lambda(m/n)(1-H(w))]^2}
\end{aligned}$$

With this result, we can derive the equilibrium employment in a firm paying the wage w

$$\begin{aligned}
\ell(w) &= \lim_{\varepsilon\to 0}\frac{n[G(w+\varepsilon)-G(w)](1-u)}{m[H(w+\varepsilon)-H(w)]} \\
&= \frac{n}{m}\frac{g(w)}{h(w)}(1-u) \\
&= \frac{n}{m}\frac{\frac{q[q+\lambda(m/n)]h(w)}{[q+\lambda(m/n)(1-H(w))]^2}}{h(w)}\frac{\lambda(m/n)}{q+\lambda(m/n)} \\
&= \frac{n}{m}\frac{q\lambda(m/n)}{[q+\lambda(m/n)(1-H(w))]^2}
\end{aligned}$$

For the firm paying the lowest wage $w = z$, we can write the employment as

$$\ell(z) = \frac{n}{m} \frac{q\lambda(m/n)}{[q + \lambda(m/n)]^2}$$

Question E

Example answer: The social welfare function is given by

$$\begin{aligned} \Omega &= n[y(1-u) + zu] - mc_f \\ &= ny\frac{\lambda(m/n)}{q+\lambda(m/n)} + nz\frac{q}{q+\lambda(m/n)} - mc_f \end{aligned}$$

The first term is the total production for the employed workers, the second term is the total home production done by unemployed, and the third term is the total firm costs of operating in the market.

The planner's first-order condition is given by

$$\begin{aligned} \frac{\partial \Omega}{\partial m} &= 0 \Leftrightarrow \\ ny\frac{\lambda'(m/n)\frac{1}{n}[q+\lambda(m/n)] - \lambda(m/n)\lambda'(m/n)\frac{1}{n}}{[q+\lambda(m/n)]^2} - nz\frac{q\lambda'(m/n)\frac{1}{n}}{[q+\lambda(m/n)]^2} - c_f &= 0 \Leftrightarrow \\ y\frac{\lambda'(m/n)q}{[q+\lambda(m/n)]^2} - z\frac{q\lambda'(m/n)}{[q+\lambda(m/n)]^2} - c_f &= 0 \Leftrightarrow \\ (y-z)\frac{q\lambda'(m/n)}{[q+\lambda(m/n)]^2} &= c_f \end{aligned}$$

Whereas the l.h.s. is the marginal benefits of an additional firm operating in the market, the r.h.s. is the fixed cost c_f. The marginal benefits consist of two parts, the (constant) gain of moving a person from unemployment to employment, $y - z$, whereas the second part is the increase in steady-state employment when an additional firm enters the market. Since $\lambda(\cdot)$ is concave, the marginal benefits of an additional firm is declining in the number of firms whereas the operating costs are fixed, so there is a unique optimal number of firms.

Question F

Example answer: In the free-entry search equilibrium, we have that $\pi(w) = 0$, that is

$$(y - w)\ell(w) = c_f$$

and evaluating this in $w = z$ and inserting $\ell(z)$ from Question D implies that

$$\begin{aligned}(y-z)\,\ell(z) &= c_f \Leftrightarrow \\ (y-z)\frac{n}{m}\frac{q\lambda(m/n)}{[q+\lambda(m/n)]^2} &= c_f \Leftrightarrow \\ (y-z)\frac{q\frac{\lambda(m/n)}{m/n}}{[q+\lambda(m/n)]^2} &= c_f\end{aligned}$$

Denoting the optimal number of firms by m^o and the free-entry search equilibrium number of firms by m^{se}, we can combine the two equations to write

$$(y-z)\frac{q\lambda'(m^o/n)}{[q+\lambda(m^o/n)]^2} = (y-z)\frac{nq\frac{\lambda(m^{se}/n)}{m^{se}/n}}{[q+\lambda(m^{se}/n)]^2}$$

Due to the concavity of the job arrival rate $\lambda(\cdot)$, we must for a given m/n have that the marginal effect on the arrival of an extra firm is less than average effect, that is $\lambda'(m/n) < \frac{\lambda(m/n)}{m/n}$. Therefore, it must be the case that the free-entry search equilibrium implies a higher number of firms than the efficient level, that is $m^{se} > m^o$.

Question G

Example answer: As argued in Question F, the search equilibrium number of firms is higher than the optimal level. Introducing a minimum wage $w_{\min} > z$ will lower the number of firms as this decreases the expected profits for a given number of firms. Therefore, by introducing a minimum wage we might get closer to the optimal number of firms whereby the congestion effect of an additional firm is lowered. More formally, as the only endogenous variable in the free-entry condition $(y - w_{\min})\frac{nq\frac{\lambda(m/n)}{m}}{[q+\lambda(m/n)]^2} = c_f$ is m, increasing $w_{\min}$ must decrease m.

Question H

Example answer: Equating the free-entry condition for the case with a minimum wage with the first-order condition for the planner's problem and evaluating this combined equation in the optimal number of firms, $m = m^o$, we can write

$$
\begin{aligned}
(y - w_{\min}) \frac{q \frac{\lambda(m^o/n)}{m^o/n}}{\left[q + \lambda\left(m^o/n\right)\right]^2} &= (y - z) \frac{q \lambda'\left(m^o/n\right)}{\left[q + \lambda\left(m^o/n\right)\right]^2} \Leftrightarrow \\
y - w_{\min} &= (y - z) \frac{\lambda'\left(m^o/n\right)}{\frac{\lambda(m^o/n)}{m^o/n}} \Leftrightarrow \\
w_{\min} &= \frac{\lambda'\left(m^o/n\right)}{\frac{\lambda(m^o/n)}{m^o/n}} z + \left(1 - \frac{\lambda'\left(m^o/n\right)}{\frac{\lambda(m^o/n)}{m^o/n}}\right) y
\end{aligned}
$$

The optimal minimum wage is a weighted average between the opportunity costs of employment, z, and the productivity, y. The more curvature of the job arrival rate of a worker, the higher is the optimal minimum wage since then the congestion effect is larger.

Part III - Paternal leave and discrimination (max 1600 words)

Question A

Example answer: Given the competitive wage setting in stage 1 of the model, firms will at most be willing to offer each worker a wage equal to the expected output that the worker will generate if hired. If we assume that firms can observe whether each worker is male or female but cannot observe whether workers want children, we can find the expected output generated by hiring a man and a woman respectively. Since men always work full time, the expected output from a man is:

$$f(1) = y$$

With probability p a woman who is hired has a child, goes on maternity leave and does not work. Otherwise she works full time. As a result, expected output from a woman is:

$$p \cdot f(0) + (1 - p) f(1) = (1 - p) y$$

Now if firms are offering any worker a wage strictly below his or her expected output, any firm can increase their wage offer slightly and be sure to hire the worker and earn a profit. In equilibrium, wage offers of each worker will therefore be bid up so that all workers are offered and accept a job offer with a wage equal to their expected output. Letting w_m be the wage of men and w_f be the wage of women we have in equilibrium:

$$w_m = y \tag{1}$$
$$w_f = (1-p)y \tag{2}$$

This result hinged on the assumption that firms can observe and condition their wage offers on gender but nothing else in stage 1. If we change this assumption and assume that firms can observe both gender and whether or not a worker wants to have children, workers will still end up being offered and accepting a wage equal to their expected output, however now the wage and expected output will depend not only on gender but also on whether the worker wants children. Since all men and those women that do not want children work full time, they will all be offered and accept a wage of $f(1) = y$. Women that do want children, however, will be offered and accept a wage of $f(0) = 0$.

Question B

Example answer: As discussed above, the case where wages differ only across gender is the case where firms can observe gender but cannot observe whether people want children. From equations 1 and 2 above we see that $w_m > w_f$. There is thus a gender gap in labor market outcomes in the sense that women earn less than men. The reason this occurs is that when firms hire a woman, they factor in that she may go on maternity leave and be less productive for the firm (in fact she will be completely unproductive).

The question of who is being discriminated against in this model depends on how one defines discrimination. Clearly, all women are being paid less than men, however, women who have children also perform less market work than men so it is not clear that this should really be called discrimination.

One group that is clearly the victim of a specific type of discrimination, however, is the group of women who do not want children. These women are being paid less than men solely because *other* women go on maternity leave. This is an example of statistical discrimination, which occurs when an employee is being treated differently solely because he or she belongs to a certain group and this causes firms to (statistically) expect them to be less attractive to hire.

Question C

Example answer: As in Question A, both men and women will be offered and accept a wage equal to their expected productivity. When hiring a man, the new policy means that with probability p he now has a child, goes on partial paternity leave and works half the time ($h = \frac{1}{2}$). Otherwise he works full time as before ($h = 1$). The expected production from hiring a man is therefore:

$$p \cdot f(\tfrac{1}{2}) + (1-p)f(1) = p \cdot y \cdot \tfrac{1}{2} + (1-p) \cdot y = y(1 - \tfrac{1}{2}p)$$

When hiring a woman, the new policy means that with probability p she has a child, goes on partial maternity leave and works half the time ($h = \frac{1}{2}$). Otherwise she works full time ($h = 1$). As is clear, this is exactly the same as for a man so the expected output from hiring a woman and a man is the same.

Since wages equal expected output we get:

$$w_m = w_f = y(1 - \tfrac{1}{2}p)$$

We see that under this policy, the gender gap in wages from before has disappeared and men and women earn the same wage. The reason for this is that by forcing men to take half the paternity leave, firms now correctly view men and women as equally likely to go on leave when they have a child. As a result, the expected output from hiring a man and woman is the same.

Question D

Example answer: In the version of the model without the new policy, there are $\frac{N}{2}$ men who always work full time and produce $f(1) = y$ each. There are also $(1-p) \cdot \frac{N}{2}$ women who do not have a child, work full time and also produce $f(1) = y$ each. Finally there are $p \cdot \frac{N}{2}$ women who have a child, go on maternity leave and do not produce. As a result, the total output is:

$$\tfrac{N}{2} \cdot y + (1-p) \cdot \tfrac{N}{2} \cdot y = N \cdot (1 - \tfrac{1}{2}p) \cdot y$$

In the version of the model with the new policy, there are $(1-p) \cdot \frac{N}{2}$ men and $(1-p) \cdot \frac{N}{2}$ women who do not have a child, work full time and produce $f(1) = y$. In addition there is $p \cdot \frac{N}{2}$ men and $p \cdot \frac{N}{2}$ women who have a child, work half time and produce $f(\frac{1}{2}) = \frac{1}{2} \cdot y$. The total output is therefore:

$$\begin{aligned}
&(1-p) \cdot \tfrac{N}{2} \cdot y \\
&+ (1-p) \cdot \tfrac{N}{2} \cdot y \\
&+ p \cdot \tfrac{N}{2} \cdot \tfrac{1}{2} \cdot y \\
&+ p \cdot \tfrac{N}{2} \cdot \tfrac{1}{2} \cdot y = N \cdot (1 - \tfrac{1}{2}p) \cdot y
\end{aligned}$$

Comparing the total output with and without the new policy, we see that they are the same. As a result, the new policy has not changed the efficiency of the outcome. The reason this happens is that the new policy has decreased the amount work done by men but increased the amount of work done by women by the same amount. The new policy has therefore changed the distribution of work across people but has not affected the total amount of work being done. Since the production technology in the model has constant returns to labor (one unit of time worked always produces y units of output), total output - and therefore efficiency - is unchanged.

In terms of whether the new policy is a good idea, the previous paragraph makes clear that in terms of economic efficiency, the policy is neither a good or a bad idea, since the policy does not affect the efficiency of the outcome. As we saw in Question C, however, the policy does have distributional consequences since it lowers the wages for men and increases the wage for women so that the gender gap in wages disappear. This makes the policy attractive if one wants to lower (gender) inequality.

Question E

Example answer: Now, in the version of the model without the new policy, there are $\frac{N}{2}$ men who always work full time and produce $f(1) = y - c$ each. There are also $(1-p) \cdot \frac{N}{2}$ women who do not have a child, work full time and also produce $f(1) = y - c$ each. Finally there are $p \cdot \frac{N}{2}$ who have a child, go on maternity leave and do not produce anything. As a result, the total output is:

$$\tfrac{N}{2} \cdot (y-c) + (1-p) \cdot \tfrac{N}{2} \cdot (y-c) = N \cdot (1 - \tfrac{1}{2}p) \cdot y - N \cdot (1 - \tfrac{1}{2}p) \cdot c$$

Similarly, in the version of the model with the new policy, there are now $(1-p) \cdot \frac{N}{2}$ men and $(1-p) \cdot \frac{N}{2}$ women who do not have a child, work full time and produce $f(1) = y - c$. In addition there is $p \cdot \frac{N}{2}$ men and $p \cdot \frac{N}{2}$ women who have a child, work half time and produce $f(\frac{1}{2}) = \frac{1}{2} \cdot y - c$. The total output is therefore:

$$\begin{aligned}&(1-p)\cdot\tfrac{N}{2}\cdot(y-c)\\&+(1-p)\cdot\tfrac{N}{2}\cdot(y-c)\\&+p\cdot\tfrac{N}{2}\cdot(\tfrac{1}{2}\cdot y-c)\\&+p\cdot\tfrac{N}{2}\cdot(\tfrac{1}{2}\cdot y-c)=N\cdot(1-\tfrac{1}{2}p)\cdot y-N\cdot c\end{aligned}$$

The change in output after the new policy is introduced is:

$$\begin{aligned}&N\cdot(1-\tfrac{1}{2}p)\cdot y-N\cdot c\\&-\big(N\cdot(1-\tfrac{1}{2}p)\cdot y-N\cdot(1-\tfrac{1}{2}p)\cdot c\big)\\&=-N\cdot\tfrac{1}{2}p\cdot c<0\end{aligned}$$

We see that with the new production function, the introduction of the new policy causes a decrease in total output so leads to a less efficient outcome. The reason for this is that with a fixed cost per-worker, there is a type of increasing returns in the per-worker production function: With the new production function, one worker working full time produces more than two workers working half time. The loss of efficiency therefore occurs because the new policy exactly causes men and women with a child to both work half time instead of one of them working full time.

Since the new policy now leads to a less efficient outcome, it is clearly not a good idea from the point of view of economic efficiency. The policy may of course still be attractive for distributional reasons since it lowers (gender) inequality, however, as opposed to in Question D, there will now be a trade off between the achieving distributional goals and economic efficiency.